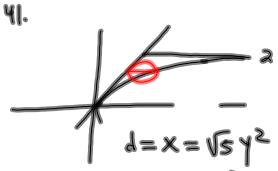
41.



$x = \sqrt{5-y^2}$

$\Delta V = \pi r^2 \Delta y$

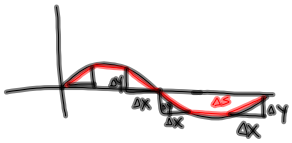
$\int_0^2 \pi \left(\frac{\sqrt{5}y^2}{2}\right)^2 dy$

8π

Dec 11-9:16 AM

7.4 Length of a smooth curve $\int_a^b f(x)\,dx = \lim \sum f(x_i)\,\Delta x$

Approximate the length of the curve y = sin(x) from x = 0 to x = 2pi



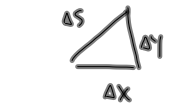
$\sum_{i=1}^{n} \Delta s_i$

$\sum \sqrt{\Delta x^2 + \Delta y^2}$

$\sum \sqrt{\left(1 + \frac{\Delta y^2}{\Delta x^2}\right)\Delta x^2}$

$\lim_{\Delta x \to 0} \sum \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\,\Delta x$

$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$



$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

$h = \Delta x$

Dec 17-5:53 PM

Definition of arclength

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$$

use for $x = g(y)$

Dec 17-6:43 PM

Find the exact length of the curve $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1, 0 \le x \le 1$

$\int_0^1 \sqrt{1+8x}\,dx$

$\frac{dy}{dx} = \frac{3}{2} \cdot \frac{4\sqrt{2}}{3} x^{1/2} = 2\sqrt{2}\sqrt{x}$

$\left(\frac{dy}{dx}\right)^2 = \left(2\sqrt{2}\sqrt{x}\right)^2 = 8x$

$\frac{1}{8} \cdot \frac{2}{3}(1+8x)^{3/2} \Big|_0^1$

$\frac{1}{12}\left(9^{3/2} - 1\right)$

$\frac{1}{12}(27-1) = \frac{26}{12} = \frac{13}{6}$

Dec 17-6:49 PM

A vertical tangent

Find the length of the curve $y = \sqrt[3]{x}$ between (-8,-2) and (8,2)

$x = y^3 \quad \frac{dx}{dy} = 3y^2$

$\frac{dy}{dx} = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$

$\int_{-8}^{8} \sqrt{1+\left(\frac{1}{3x^{2/3}}\right)^2}\,dx \overset{!}{=} 17.2606$

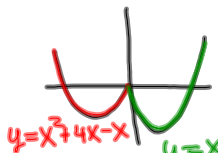
$\int_{-2}^{2} \sqrt{1+(3y^2)^2}\,dy = 17.2607$

Dec 17-6:59 PM

A cusp

$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Find the length of the curve $y = x^2 - 4|x| - x$ from x=-4 to x=4



$\int_{-4}^{0} \sqrt{1+(2x+3)^2}\,dx + \int_{0}^{4} \sqrt{1+(2x-5)^2}\,dx$

$y = x^2+4x-x$

$y = x^2+3x$

$y = x^2-4x-x$

$y = x^2-5x$

Dec 17-7:16 PM

Jan 3-12:44 PM